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October 31, 1979

This is the sixth quarterly status report on a program for Image Understanding Using Overlays, conducted by Westinghouse for UMd under Contract DAAG53-76-C-0138 with the U.S. Army Mobility Equipment Research and Development Command, Ft. Belvoir, Virginia 22060.

Prepared for

Computer Science Center  
University of Maryland  
College Park, Maryland 20742

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By

Westinghouse Defense and Electronics Systems Center  
Systems Development Division  
Baltimore, Maryland 21203

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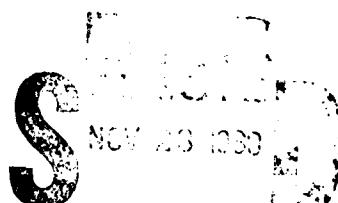
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## INTRODUCTION

This is the sixth quarterly status report on a program to implement higher level image processing algorithms, being conducted by the Westinghouse Systems Development Division for the Computer Science Center, University of Maryland. Support for the program is provided by the Defense Advanced Research Projects Agency (DARPA) under contract DAAG53-76-C-0138 with the U.S. Army Mobility Equipment Research and Development Command.

The report was prepared by Arden Helland of Westinghouse, with contributions by Shmuel Peleg and Azriel Rosenfeld of the University of Maryland. The Westinghouse Program Manager is Dr. Glenn Tisdale. The work was discussed at monthly meetings, held at the University of Maryland (UMD) with Professor Azriel Rosenfeld of UMD and Dr. George Jones of NVEOL.

This report covers results of special analysis performed as part of the recent work to support the UMD in the statistical testing of complex algorithms. The planned steps in this support program are:

Selection of processing algorithms for evaluation.

Analysis and adaptation of algorithms for execution

on the Programmable Array Processor (PAP).

Evaluation of algorithms on a PDP-VAX GP computer.

Throughput analysis; PAP vs. VAX.

Processing of a set of imagery.

Results are reported at regular intervals as appropriate.

This report analyzes the segmentation properties of gray level relaxation applied to the two-label case. The results produce threshold, speed and stability criteria to facilitate subsequent processing. Most of these results are experimentally verified in a report by Azriel Rosenfeld and Russel C. Smith. Evaluation and verification is in

progress at Westinghouse using comparable imagery and test patterns. The current status of the computer program modeling and test results will be covered in a separate report to be completed shortly. Efforts underway and planned for the immediate future include evaluation of image samples on the VAX, throughput analysis for the PAP, and relaxation processing for multiple label cases. The analysis and subsequent testing is intended to apply to monochrome TV or FLIR imagery (hence, one-dimensional data) and usually only one object polarity (two labels) with extension to both object polarities in the same imagery (three labels).

## ABSTRACT

This report analyzes the segmentation properties of the Peleg relaxation scheme for the one-dimensional, two-label case. It is shown that if the probability of either label is identically zero, then that label probability will remain identically zero for all iterations. However, if the label probabilities are non-zero, then the label probability at each iteration will either increase toward unity or decrease toward zero, depending on whether the average probability of the neighborhood is above or below a threshold determined by the relationship of the compatibility coefficients. Further analysis of the boundaries between regions of different labels shows that boundary stability requires that the net effective coefficients for both labels be equal. In addition, it is shown that the speed at which ambiguity is resolved is maximized if non-zero compatibility coefficients are allowed for alike labels. There has been no indication that increased speed alters the results. Extensions to multiple-label cases are discussed.

## I. Relaxation

Relaxation labeling has been shown to provide reduced error rates in pixel classification compared to procedures that are based solely on local evidence. Relaxation classification uses initial label probabilities, which may be modified based on the label probabilities in the neighborhood (region) of each decision point (pixel). The objective for the cases of interest is to segment darker regions from lighter regions in the presence of significant noise which causes ambiguity within the regions. It has been shown by several examples that for light/dark labels, darker regions tend to become black, and lighter regions become white as the relaxation iterations progress. However, for the cases cited, the progress toward unambiguous labels is rather slow, the boundaries between the regions do not appear to be stationary, and changing the coefficients alters the results. The process appears to have the intriguing capability of making a very faint target in selected windows appear obvious after a few iterations. It therefore appears that relaxation simultaneously provides the advantages of classification by regional (vs local) evidence as well as the contrast of segmentation (but without selecting a threshold). It has been recognized that in many cases relaxation results appear to degrade after several iterations. Also, the computational complexity of relaxation constrains the number of iterations to the minimum number practical. Therefore, best efficiency is obtained if the desired results are obtained with a minimum number of iterations. Eventual convergence to an undesirable limit, however, may not be a serious disadvantage in practical applications.

## II. Compatibility Coefficients

The compatibility coefficients used in relaxation may be derived in several ways. Peleg proposed that they be derived by the relationship of joint to individual probabilities. These may be derived from a statistical model for the problem domain, or may be estimated from a statistical estimate using representative imagery. Some of the most intriguing relaxation results have been shown with the coefficients derived solely from the sample window to be processed; in most of these cases the sample windows were manually selected to include a known object.

It has been stated that for many cases of light/dark relaxation, light reinforces light and dark reinforces dark. This implies that the coefficients should be zero for all relationships except for light/light and dark/dark. If a light/dark or a dark/light coefficient is non-zero, this implies that dark also reinforces light (and light also reinforces dark). This will tend to reduce the effect of the surrounding region, so that each point is changed less than if only the alike labels are allowed to interact at each iteration. If, however, the unlike coefficients were allowed to become greater than those for the alike labels,

the result should be a tendency for points to be assigned labels opposite to the surrounding region.

The form of the coefficients suggested by Peleg is:

$$r_{ij}(\alpha, \beta) = \frac{P(\alpha, \beta)}{P(\alpha) P(\beta)}$$

Now, the joint probability  $P(\alpha, \beta)$  can be expressed as a function of the conditional probability as follows:

$$P(\alpha, \beta) = P(\alpha) \cdot P(\beta/\alpha)$$

so that  $r_{ij}(\alpha, \beta) = \frac{P(\alpha) \cdot P(\beta/\alpha)}{P(\alpha) \cdot P(\beta)} = \frac{P(\beta/\alpha)}{P(\beta)}$

Now, to consider what values may be represented by the above. First, consider an idealized case; if all alike labels are highly clustered then  $P(\alpha/\alpha) \rightarrow 1$  and correspondingly,  $P(\beta/\alpha) \rightarrow 0$  for  $\beta \neq \alpha$ . Also,  $P(\beta) \approx P(\alpha) \approx .5$  if both label classes are equally likely (or equally represented in a statistical sample). Therefore, it may be deduced that  $r_{ij}(\alpha, \alpha)$  and  $r_{ij}(\beta, \beta)$  both approach a value of 2.0 for such an idealized case. This implies that ideal coefficients should be equal for alike labels. If region boundaries are considered,  $P(\beta/\alpha)$  and  $P(\alpha/\beta)$  would be small and approximately equal because each boundary is counted twice - once as  $\alpha \rightarrow \beta$  and again as  $\beta \rightarrow \alpha$ . Therefore, ideal coefficients should have very low values if reinforcement of boundaries is not intended.

Second, consider what happens in the idealized case when  $\alpha$  and  $\beta$  labels are not equally represented. If we consider sample frames dominated by class  $\beta$ , then as  $P(\beta) \rightarrow 1$ , the idealized value of  $r_{ij}(\beta, \beta)$  is reduced from 2.0 toward 1, but the corresponding result is that as  $P(\alpha) \rightarrow 0$ , the  $r_{ij}(\alpha, \alpha)$  increases without limit. This will provide much greater reinforcement for the scarce label than for the more common label.

Third, consider the effects of random noise added to a sample frame which contains clustered regions. As the noise becomes more significant, the alike labels within the regions become less correlated, so that  $P(\alpha/\alpha)$  and  $P(\beta/\beta)$  are reduced from 1.0 toward 0.5. Correspondingly, for the unlike labels,  $P(\beta/\alpha)$  and  $P(\alpha/\beta)$  increase from 0 toward 0.5. Therefore, we may deduce that noise which causes ambiguity in a sample used for deriving coefficients will, in turn, cause ambiguity in the relationship of the compatibility coefficients. If these coefficients are, in turn, used to process a noisy image, the ambiguity will have a tendency to be preserved longer than if lower unlike coefficients were used. However, if the random noise does not affect the proportional representation of the two labels, the relative effect of an unequal representation is unchanged.

### III. Threshold Analysis

The following analysis is based on the probability updating rule developed by Peleg for two labels. The two labels will be called b and w, so that  $P_{jb}$  is the probability that the ith pixel is black. Likewise,  $P_{jb}$  is the probability that the jth neighbor is black. In the general case, the compatibility coefficients are a function of the orientation of the jth neighbor to the ith pixel, as well as their labels. This report makes the assumption that the compatibility coefficients are independent of (or averaged over) the orientation, so that it is a function only of the labels, as follows:

Assumption:  $r_{ij}(\alpha, \beta) \rightarrow r_{\alpha\beta}$

For labels b (black) and w (white), therefore, we have  $r_{bb}$ ,  $r_{bw}$ ,  $r_{ww}$ , and  $r_{wb}$ .  $r_{bb}$  and  $r_{ww}$  are called the alike label coefficients;  $r_{bw}$  and  $r_{wb}$  are the unlike label coefficients.

The updating rule consists of the following parts where K is the iteration index and N is the number of neighbors:

$$S_b = P_{ib} \sum_{j=1}^K P_{jb} \cdot r_{bb} + P_{ib} \sum_{j=1}^N P_{jw} \cdot r_{bw}$$

$$S_w = P_{iw} \sum_{j=1}^K P_{jw} \cdot r_{ww} + P_{iw} \sum_{j=1}^N P_{jb} \cdot r_{wb}$$

These are used to generate probabilities for the next iteration as follows:

$$P_{ib}^{K+1} = \frac{S_b}{\frac{K}{S_b} + \frac{K}{S_w}} \quad \text{and} \quad P_{iw}^{K+1} = \frac{S_w}{\frac{K}{S_b} + \frac{K}{S_w}}$$

$S_b^K$  and  $S_w^K$  may be reduced to two variables by making the following substitution:

$$\hat{P}_\alpha = \frac{1}{N} \sum_{j=1}^N P_{j\alpha}$$

$$\hat{P}_w = 1 - \hat{P}_b$$

Also, for simplicity, the iteration index,  $K$ , will be dropped; where  $K+1$  occurs it will be indicated by \*. Therefore, by multiplying by the number of neighbors  $N$  and combining terms, we have

$$\begin{aligned} S_b &= P_{ib} (\hat{P}_b (r_{bb} - r_{bw}) + r_{bw}) \equiv P_{ib} \cdot F_b \\ S_w &= (1 - P_{ib}) ((1 - \hat{P}_b) (r_{ww} - r_{wb}) + r_{wb}) \\ &= (1 - P_{ib}) (\hat{P}_b (r_{wb} - r_{ww}) + r_{ww}) \equiv (1 - P_{ib}) \cdot F_w \end{aligned}$$

Now the change in the black probability of the  $i$ th pixel for each iteration is defined as follows:

$$\Delta P_{ib}^* = P_{ib}^* - P_{ib} = \frac{S_b}{S_b + S_w} - P_{ib} = \frac{S_b - P_{ib}(S_b + S_w)}{S_b + S_w}$$

$$= \frac{P_{ib} \cdot F_b - P_{ib} (P_{ib} \cdot F_b + (1-P_{ib})F_w)}{P_{ib} \cdot F_b + (1-P_{ib})F_w}$$

$$= \frac{P_{ib} (F_b - F_w)(1-P_{ib})}{P_{ib} \cdot F_b + (1-P_{ib})F_w}$$

It may be noted that if the denominator is positive, then the behavior of  $\Delta P_{ib}$  is determined by the numerator. The denominator must be non-zero for all normal cases; the only condition that would permit both  $S_b$  and  $S_w$  to be zero is if  $P_{ib}$  and  $\hat{P}_b$  have absolutely certain but opposite labels.

Now, the product of the two factors involving  $P_{ib}$  is:

$$P_{ib} (1-P_{ib})$$

This is a quadratic function that is zero for both  $P_{ib} = 0$  and  $P_{ib} = 1$ ; this causes  $S_b$  or  $S_w$  to be zero, respectively for these two conditions. The product is positive for all values within probability space  $0 < P_{ib} < 1$ , with a maximum value occurring at  $P_{ib} = .5$ . Therefore, the behavior of  $\Delta P_{ib}$  depends on the  $F_b - F_w$  term as follows:

$$F_b - F_w \geq 0 \Rightarrow \Delta P_{ib} \geq 0$$

Now, we may solve for  $F_b - F_w \geq 0$  as a function of  $\hat{P}_b$  and the compatibility coefficients as follows:

$$\begin{aligned} F_b - F_w &\geq 0 \\ \hat{P}_b (r_{bb} - r_{bw}) + r_{bw} - \hat{P}_b (r_{wb} - r_{ww}) - r_{ww} &\geq 0 \\ \hat{P}_b (r_{bb} - r_{bw} + r_{ww} - r_{wb}) &\geq r_{ww} - r_{bw} \\ \hat{P}_b &\geq \frac{r_{ww} - r_{bw}}{r_{ww} - r_{bw} + r_{bb} - r_{wb}} \equiv T_b \end{aligned}$$

Therefore, if the average black probability of the neighborhood ( $\hat{P}_b$ ) is above the threshold  $T_b$ , the black probability of the  $i$ th center pixel will be increased, regardless of its prior value if non-zero. If  $\hat{P}_b$  is identically equal to  $T_b$ ,  $P_{ib}$  is unchanged. Conversely, if  $\hat{P}_b$  is below  $T_b$ ,  $P_{ib}$  is decreased. Therefore,  $T_b$  represents a point of divergence, or conditional stability; any perturbation that moves the neighborhood probability away from the threshold will change  $P_{ib}$  in the same direction.

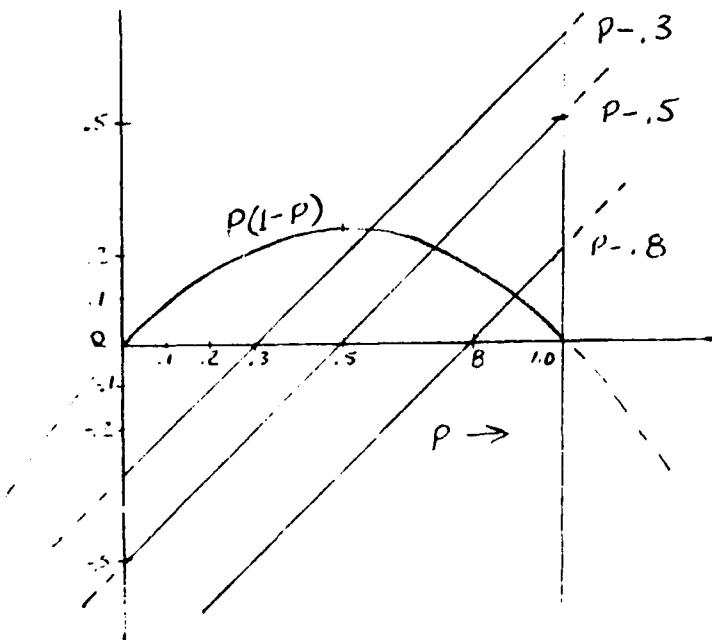
The above function of  $\hat{P}_b$  is, of course, a linear increasing function. If plotted, it crosses the 0 axis at  $T_b$  and reaches its maximum negative and positive values at the constraint limits defined by:  $0 \leq P_b \leq 1$ . Various values of  $T_b$  would define a family of lines with unity slope, intersecting the 0 axis at  $T_b$ .

We may, therefore, make the following conclusions regarding the response of  $\Delta P_{ib}$  to  $P_{ib}$  and  $\hat{P}_b$ .

1. For lighter regions:  
 $P_{ib} < 1$  and  $\hat{P}_b < T_b \Rightarrow P_{ib}^* < P_{ib}$
2. For darker regions:  
 $P_{ib} > 0$  and  $\hat{P}_b > T_b \Rightarrow P_{ib}^* > P_{ib}$
3. If  $\hat{P}_b < T_b$  for all iterations,  $P_{ib}^* \rightarrow 0$
4. If  $\hat{P}_b > T_b$  for all iterations,  $P_{ib}^* \rightarrow 1$

Therefore,  $P_{ib} = 0$  and  $P_{ib} = 1$  are values toward which all pixels within consistent regions will converge. Although  $\hat{P}_b$  may change from one iteration to the next, it is unlikely to change its relationship to the threshold (except if it is very near the threshold or if the initial label probabilities are inconsistent).

A sketch of  $P(1-P)$  and a few cases of  $P - T$  is shown below:



#### IV Speed Analysis

Inspection of the expressions for  $S_b$ ,  $S_w$  and the subsequent definition for  $\Delta P_{ib}$  show that the unlike coefficients  $r_{bw}$  and  $r_{wb}$  appear to have a tendency to partially counteract the relaxation process. Therefore, it is of interest to determine how  $\Delta P_{ib}$  varies as a function of the compatibility coefficients. As derived previously,

$$\Delta P_{ib} = \frac{P_{ib} (1-P_{ib}) (F_b - F_w)}{P_{ib} \cdot F_b + (1-P_{ib}) F_w}$$

Now,  $P_{ib}$  defines the difference between the current probability of black and the absolute white probability; likewise  $1-P_{ib} = P_{iw}$  defines the difference from the absolute black probability. Therefore, it is convenient to define relative speed as the ratio of the change in probability to the remaining differences from the appropriate absolute label. Thus, the relative speed of convergence to the black label is  $C_b = \Delta P_{ib}/(1-P_{ib})$ . Since  $-\Delta P_{iw} = \Delta P_{ib}$ , the relative speed of convergence to the white label is likewise defined as  $C_w = -\Delta P_{ib}/P_{ib}$ .

$$\text{Therefore, } C_b = \frac{\Delta P_{ib}}{1-P_{ib}} = \frac{P_{ib} (F_b - F_w)}{P_{ib} \cdot F_b + (1-P_{ib}) F_w}$$

It can be shown that  $C_b$  is maximum with respect to  $P_{ib}$  as it approaches the constraint limit of unity. Likewise,  $C_b$  is maximum with respect to  $\hat{P}_b$  when it also approaches the constraint limit of unity. Therefore, if  $C_b$  is evaluated at  $P_{ib} = \hat{P}_b = 1$ :

$$C_b = \frac{F_b - F_w}{F_b} = \frac{(r_{bb} - r_{bw}) + r_{bw} - r_{ww} - (r_{wb} - r_{ww})}{r_{bb} - r_{bw} + r_{wb}}$$

$$C_b = \frac{r_{bb} - r_{wb}}{r_{bb}}$$

Likewise,  $C_w$  is evaluated at  $P_{ib} = \hat{P}_b = 0$ :

$$C_w = \frac{-\Delta P_{ib}}{P_{ib}} = \frac{F_w - F_b}{F_w} = \frac{r_{ww} - r_{bw}}{r_{ww}}$$

These expressions for maximum speed of convergence show that non-zero unlike coefficients directly reduce speed. It is also of interest to evaluate the speed for the case where the probability label for the  $i$ th (center) pixel is not equal to the average of the neighbors. For example, if  $P_{ib} = .5$ , (a typical ambiguous level) and  $\hat{P}_b = 1$ , then

$$C_B(5) = \frac{F_b - F_w}{F_b + F_w}$$

$$C_B(.5) = \frac{(r_{bb} - r_{bw}) + r_{bw} - r_{ww} - (r_{wb} - r_{ww})}{(r_{bb} - r_{bw}) + r_{bw} + r_{ww} + (r_{wb} - r_{ww})} = \frac{r_{bb} - r_{wb}}{r_{bb} + r_{wb}}$$

It may be noted that this value is similar to the  $P_{ib} = 1.0$  case except for a reduction due to the addition of the unlike term in the denominator. This indicates that the resulting probability for the  $i$ th pixel is strongly determined by its neighborhood if the neighborhood label is nearly certain. Further, if the unlike coefficients are zero and the neighborhood label is certain then  $C_B = 1.0$ . This means that the  $i$ th pixel label will be driven to the same certain label in one iteration, independent of its previous probability (if non-zero, of course).

This result can be considered as a limiting condition that has equivalence to post-processing. If the center pixel is at least somewhat ambiguous (non-zero probability) then it will be made to agree with its neighbors if all of them are in agreement and have essentially certain labels. However, the effect on the center pixel will be reduced if not all of the

neighbors agree because the average of the neighbors is used to modify the center pixel. This suggests that there is a similarity between majority-voted post-processing and the unity alike coefficient case of relaxation, as follows: If at least a majority of the neighborhood has essentially certain labeling, then the neighborhood average will be above the threshold of one-half, and the center pixel probability will be increased. If the region surrounding that neighborhood also has a majority with the same label, then the neighborhood average will remain above the threshold for further relaxation iterations, so the center pixel probability will continue to increase. This applies to any pixel surrounded by a majority of neighbors with a specific label. Therefore, any region containing a majority of a specific label will converge to uniform labeling. Therefore, this case of relaxation may be considered a probability weighted and majority-weighted region smoothing process. The major difference provided by relaxation is that if the probabilities are unambiguous or the neighborhood labels evenly divided, the result remains ambiguous until further iteration provide information from a larger surrounding region to tip the balance toward one of the labels.

The above consideration, of course, required avoidance of the case of the center pixel probability identical to zero because this prevents any change in probability. It is of interest to consider the characteristics of the apparent contradiction between zero probabilities remaining zero and probabilities changed to agree with the neighbors if their labels are absolutely certain. First, it should be realized that this case is not possible in real-world imagery because it implies absolute certainty

regarding opposite states for adjacent pixels. This requires infinite frequency bandwidth, zero noise (from all sources including thermal and quantum physics effects) and no measurement uncertainty. Second, if we substitute these conditions into the original updating function, we find that the new probabilities and  $\Delta P_i$  are undefined because the denominator is zero (for the case of the function for black probabilities, both  $P_{ib}$  and  $F_w$  in zero). Third, it seems obvious that if opposite local labels are not allowed as initial conditions, then they cannot occur as a result of smoothing or any intermediate relaxation iterations because these processes tend to reduce local variance. For example, for the unity alike coefficient case, if the center pixel probability is one-half, it will be set equal to the neighborhood average in one iteration.

Because the previous threshold and speed analysis is not valid for the case of opposite local labels, it is convenient to consider the original updating formula for the unity alike coefficient case;  $F_b$  is simply  $\hat{P}_b$ ; likewise  $F_w$  is  $\hat{P}_w$ . Now, if we consider the more general case of "opposite" labels by assuming that  $\hat{P}_b = 1 - P_{ib}$ , we can exclude the absolutely certain cases of  $\hat{P}_b = 0$  and  $P_{ib} = 0$ . If we substitute  $1 - P_{ib}$  for  $\hat{P}_b$  and, correspondingly,  $P_{ib}$  for  $1 - \hat{P}_b$ , then  $S_b = P_{ib}(1 - P_{ib})$  and  $S_w = (1 - P_{ib}) P_{ib}$ . Because  $S_b = S_w$ , the new center pixel probability will be one-half, regardless of how certain the "opposite" labels were. This result leads to the following conclusions:

1. Means should be provided to avoid absolute zero probabilities at least on input data, if relaxation is to be allowed to operate for all conditions.
2. The use of a minimum non-zero value should have no significant effect on results because the above analysis indicated performance essentially independent of absolute value as opposite label

probabilities approach certainty; previous analysis showed rapid convergence for consistent label probabilities in a region.

3. The relaxation response to conflicting labels between a center pixel and its neighborhood may be stated more generally in terms of relative label certainty as follows: If the neighborhood label is at least as certain as the center pixel is for the other label, then the center pixel probability for the neighborhood label will be increased to at least one-half. As the neighborhood certainty increases relative to the center pixel, the center pixel probability will approach certainty.

The form of the expressions for speed of convergence and threshold may be simplified somewhat if we make the following assumptions to define new variables:

$$rwb = rbw \equiv rx$$

$$rbb - rx \equiv Rb$$

$$rww - rx \equiv Rw$$

then  $Cb = Rb/rbb$

$$Cw = Rw/rww$$

$$Tb = \frac{Rw}{Rw+Rb} = \frac{1}{1+Rb/Rw}$$

$$Tw = \frac{Rb}{Rb+Rw} = \frac{1}{1+Rw/Rb}$$

The thresholds, of course, are not independent functions because  $Tb+Tw = 1$ . The above functions define speed of convergence and threshold for relaxation as a function of the selected compatibility coefficients. As shown above, the threshold is determined by the ratio of the net coefficients  $Rb$  and  $Rw$ . For the case of equal net coefficients, the threshold is  $1/2$ . For an unequal example, if the net black coefficient is twice that for white, the black threshold is only  $1/3$ .

The speed of convergence, however, is directly determined by the ratio of the net coefficient to the alike coefficient. The speed of convergence toward each label may be different; convergence to the label with the highest net coefficient will tend to be faster than the others. The speed of convergence to either label is reduced as the unlike coefficients become larger compared to the alike coefficient. For example, if the unlike coefficient is one-half of the alike coefficient, the speed of convergence measure is 50%. If the unlike coefficient is 90% of the alike coefficient, then the net coefficient is only 10% of the alike coefficient, for a speed of convergence measure of 10%. It is obvious that the maximum speed of convergence to both labels is obtained if all unlike coefficients are set to zero.

## V. Stability Considerations

The dynamic results of iterations of the relaxation process can be inferred by consideration of the preceding analysis. As previously noted, permitting non-zero unlike coefficients has a significant effect on the speed of convergence to either label. However, there is also little change when the neighborhood probability is near the threshold. This represents a conditionally stable situation; what relaxation performs is interaction between each pixel and its neighbors, then their neighbors, etc. The probabilities are adjusted wherever and whenever the neighborhood is different from the threshold. Usually, there will be some neighboring condition that will tend to unbalance the probabilities away from the threshold in one direction, causing the entire neighborhood to converge to the label determined by the dominating conditions. Assuming that there are significant regions with each label, then there will be areas that form boundaries between these regions. It appears rather obvious that a neighborhood centered on a linear boundary between two regions that have converged to probabilities of essentially zero and one will have an average probability,  $\hat{P}$ , of 0.5. However, if the threshold is also 0.5, the probability of the center pixel will be unchanged, thereby preserving the balance between black and white at the boundary. Therefore, a threshold of 0.5 is called a stable threshold. Conversely, though, if the threshold is not 0.5, then the probabilities in the boundary region will be driven toward one of the labels, forming a new boundary. Therefore, the boundary will continue to move toward the region whose label has the higher threshold. Provided that no probabilities are allowed to become identically zero, it appears that regions whose label has the lower threshold will expand without limit. This will gradually

destroy the shape of the region and if carried to a sufficiently large number of iterations would fill the entire image frame. Therefore, thresholds not equal to 0.5 are called unstable because the information contained in the original image is eventually lost (although convergence to a known label, determined by the compatibility coefficients is actually another "stable" result).

If the boundary is convex instead of linear, then the average probability of the neighborhood will be slightly biased toward the "outer" region. Therefore, a convex surface (e.g. a "corner") will tend to be straightened and a relatively small region surrounded by its opposite label will tend to shrink in size and lose some of its shape and edge detail after several iterations.

## VI RESULTS

Although the format of the Peleg relaxation process is strictly based on the sum of neighboring probabilities, allowing unlike coefficients is equivalent to an additive constant. This constant tends to retard any change in the probability of that center pixel. This definitely suggests that there is an equivalence between the "Peleg process" and the "Hummel-Zucker" process, which explicitly includes a constant. This approximate equivalence was illustrated by Smith, who processed a frame with several variations of compatibility coefficients.

To develop the equivalence of the Hummel-Zucker process, consider that its updating formula includes a unity term which is added to the average of the neighborhood probabilities.

Therefore, the unity term is equivalent to a  $1/N$  term added for each neighboring pixel. If the Hummel-Zucker coefficients are defined as  $ca(\alpha)$  for alike labels and  $cx$  for unlike labels, then the updating formulas are equivalent if  $ra(\alpha) - rx = ca(\alpha) - cx$  and  $rx = 1/N + cx$ . The equivalent  $ra(\alpha)$  and  $rx$  (based on  $N=8$ ) are the following:

$$ra(\alpha) = .125 + ca(\alpha) \quad \text{and} \quad rx = .125 + cx$$

Because the Hummel-Zucker unlike coefficients are negative, this reduces the equivalent unlike coefficient, while the alike coefficient is increased by the constant. Therefore, this will tend to result in faster convergence than the Peleg process under similar conditions. This is confirmed by computing the threshold and speed of convergence measures based on the preceding analysis, and by observation of Smith's experimental results (UMD TR.795). The following table shows the equivalent coefficients, light threshold and speed of convergence to dark for 5 sets of results for the "dark tank picture" evaluated by Smith.

Also shown are the equivalents for the unity alike coefficient case, which was previously described by A. Hanson and E. Riseman (Computer Vision Systems, Academic Press, NY, 1978).

<u>Process</u>	<u>Fig.</u>	<u>rx</u>	<u>rd</u>	<u>Rd</u>	<u>r1</u>	<u>R1</u>	<u>T1</u>	<u>Cd</u>
Hummel-Zucker	6	.107	.173	.067	.131	.024	.74	.39
Peleg	10	.91	1.27	.36	1.02	.11	.77	.28
(stable case)	11	.80	1.20	.40	1.20	.40	.50	.33
(faster case)	12	.20	1.80	1.60	1.80	1.60	.50	.89
(test case)	13	.85	1.20	.35	1.10	.25	.58	.29
(unity alike)	--	0.0	1.00	1.00	1.00	1.00	.50	1.00

These values for threshold and speed of convergence confirm what is observable in the published results. As shown, the threshold for the Hummel-Zucker case is quite similar to that for the Peleg process; the figures show that about the same object region is extracted. However, the speed for the Peleg case is noticeably slower; iteration 8 appears to be approximately equivalent to iteration 5 for Hummel-Zucker. Figures 11 and 12 represent two stable cases, with speed the primary difference. The difference in the results is quite dramatic; the final result in Figure 11 is roughly equivalent to iteration 2 of Figure 12. In comparison to Figure 6, the effect of the lower light threshold (higher dark threshold) is clearly shown by the smaller region extracted for the dark object. Figure 13 represents an intermediate case with a threshold lower than midway between the Hummel-Zucker case of Figure 6 and the stable case of Figure 11. However, the speed is nearly as slow as for the original Peleg case of Figure 10. The final result of Figure 13 indicates results roughly comparable to iteration 4 or 5 of Figure 6, but with a somewhat smaller region extracted due to the lower threshold.

The nature of the relaxation processing is visually indicated by investigation of the changes in the gray scale histogram during the relaxation iterations. The thresholds shown in the previous table were given in terms of the light probability, so the light threshold value is the proportion of the distance between extremes of the histogram with increasing light probability from left to right. Because regions to the left of threshold move towards the left extreme (and vice versa), there should be a movement away from the threshold region in both directions. However, since movement is very slow near the threshold, separation away from the threshold is not obvious for the initial iterations. As the ambiguity is resolved, a "valley" appears in the threshold region; the histogram approaches the two limits. However, if label probabilities are not allowed to reach

identically zero and the thresholds are not equal, then there will be a continued motion across the threshold. For the tank pictures with a light threshold above .5, there will be continued motion across the threshold region from right to left. This means that the larger peak in the histogram becomes smaller, increasing the height of the peak at the opposite end. If the process is carried to its limit, all pixels will end up at the end of the histogram opposite extreme. Since the histograms for the figures in the Smith report are normalized to the highest value, this trend is indicated by the increase in height of the left extreme peak. Figure 6 shows a good example of this trend. If, however, the thresholds are equal, then there is no motion across the threshold and the histogram converges to the two extremes. This trend is indicated by Figure 12.

### Conclusions

Boundary stability is related to the relaxation thresholds. If the threshold for one of the labels is less than for others, then regions with that label will tend to expand. However, small regions tend to shrink due to convex boundaries. This appears to imply that for large regions of interest, most shape detail will be preserved with (nearly) equal thresholds for each label. However, the response to small regions of one label may be improved by a moderate reduction in that threshold. The expansion due to unequal thresholds may be used to counteract the tendency for small regions to shrink.

The analysis of speed showed that the change in probability of the center pixel is reduced by the proportion of the unlike coefficient to the alike coefficient for each label. Maximum speed is obtained when unlike coefficients are zero. Also, it is shown that if the probability for the center pixel is zero for either label, then that probability cannot change as a result of further relaxation iterations. Therefore, input data should avoid zero probability to permit full operation of the relaxation functions.

When the average probability of the neighborhood is equal to (or nearly equal to) the threshold, then the probability of the center pixel is unchanged (or changed only slightly). If gray levels near the probability threshold are considered as ambiguous levels, then little change occurs until the ambiguity becomes resolved by further iterations. In other words, the segmentation process inherent in relaxation proceeds very cautiously as long as the neighborhood labeling remains ambiguous, deferring commitment to any label until information from more distant regions resolves the ambiguity. However, if the neighborhood labeling is not near the probability threshold, then the label probabilities are driven toward zero and unity at a rate

that is maximized if the unlike coefficients are zero.

The relative adjustment increases as the neighborhood approaches absolute label probabilities; this allows the process to converge rapidly when proper compatibility coefficients are chosen. Because the probability adjustment is based primarily on the average neighborhood probability, it is a smoothed parameter that should result in a much lower error rate than ordinary segmentation, even with prefiltering.

If we consider the smoothed neighborhood probability used for the threshold criterion, the slow adjustment near ambiguous labels and the rapid convergence to absolute labels, the following description of relaxation processing may be appropriate:

"A smoothing, cautious, rapidly converging segmentation labeling process"

Because relaxation is a segmentation process with a specific threshold, selection of the threshold relative to the proper gray level remains essential to obtaining the desired results. The previous analysis strongly indicates that stable operation with fast convergence requires that the matrix of compatibility coefficients be essentially an identity matrix so that thresholds are equal for both classes. This suggests that the gray scale data should be shifted by additive and/or multiplicative factors so that the desired gray level results in a probability at the threshold. One such a method was suggested by Smith; another might be to select constants such as  $M$  and  $B$  so that  $M + B \cdot Z_i = T$  where  $Z_i$  = selected gray threshold.

There is no apparent need to normalize the probabilities so that the lowest value occurring in each image frame will have probability zero and the highest value will have probability unity. The only apparent requirement is that the lowest possible value for any label should result in a positive probability. The gain and convergence properties of relaxation assure that the probabilities resulting from relaxation iterations will approach zero and unity, except possibly those regions near the probability threshold.

Although the initial normalization and coefficient selection (as analyzed by Danker and Smith) show intriguing results for a few frames this seems to be due primarily to the fact that all frames contained both light and dark classes. Therefore, that process cannot be expected to avoid false alarms if both classes are not effectively represented in a frame used to develop the normalization and compatibility coefficients. Any process which selects a new threshold from each frame to be processed seems certain to extract "something" from nearly every frame. Although selecting thresholds inversely proportional to likelihood of occurrence may seem to provide a degree of "fairness" (the lesser becomes greater), it will cause high false alarms on ambiguous data. Furthermore, processing with unequal coefficients can be expected to have high false alarm rates if the background contains any regions above the selected thresholds, because they will expand. Of course, any process based on gray level segmentation can be expected to have high false alarms if the image contains significant regions with size and gray levels undistinguishable from the desired classes of objects.

Although this report is based on the two label case for simplicity, extension to multiple labels seems to be relatively straightforward. Particularly when the labels represent relatively independent states defined in one dimension, there seems to be no significant justification for allowing compatibility coefficients for unlike labels. Stable boundaries, greatest effect per iteration and computational simplicity all require that approximately equal coefficients should be used for alike labels and iteration between independent labels need not be considered. This is equivalent to stating that the compatibility coefficients among the labels should be approximately an identity matrix. Normalization across all labels provides the gain to produce convergence to absolute labels and assures that values remain defined within probability space. The input data (gray level in this case)

will have to be appropriately scaled before transformation to probabilities for each state so that the desired input levels will correspond to the thresholds between regions.

